Business Mathematics, Logical Reasoning & Statistics

CA VINOD REDDY

calculator Tricks on page.No.19

1 DAY OR DAY 1

YOU DECIDE

CA VINOD REDDY

You can LEARN soemthing NEW Everyday, if you LISTEN!

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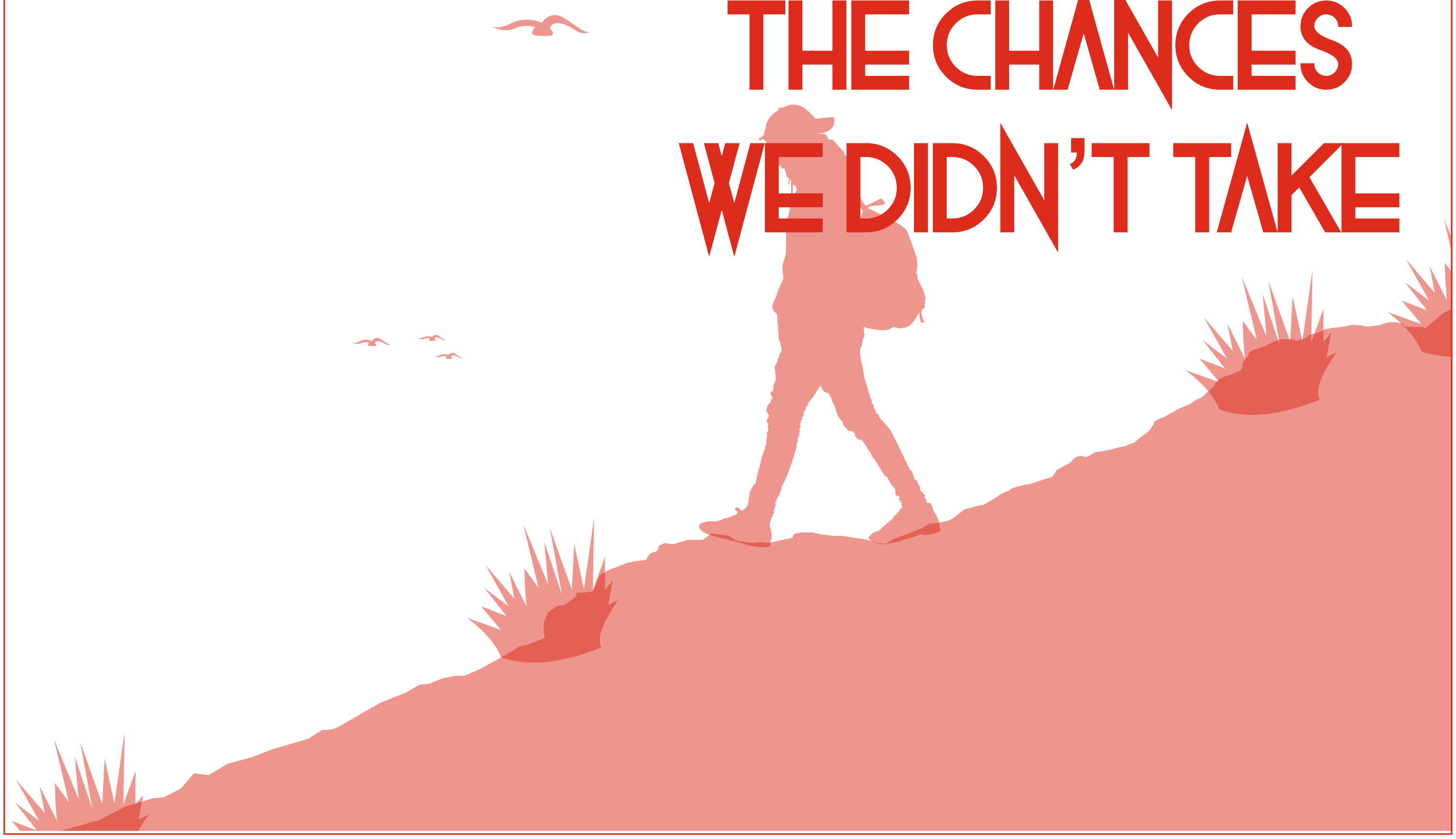
IN THE END,
WE ONLY REGRET
THE CHANCES
WE DIDN'T TAKE



Chapter 1

RATIO I PROPORTION LOGS & INDICES

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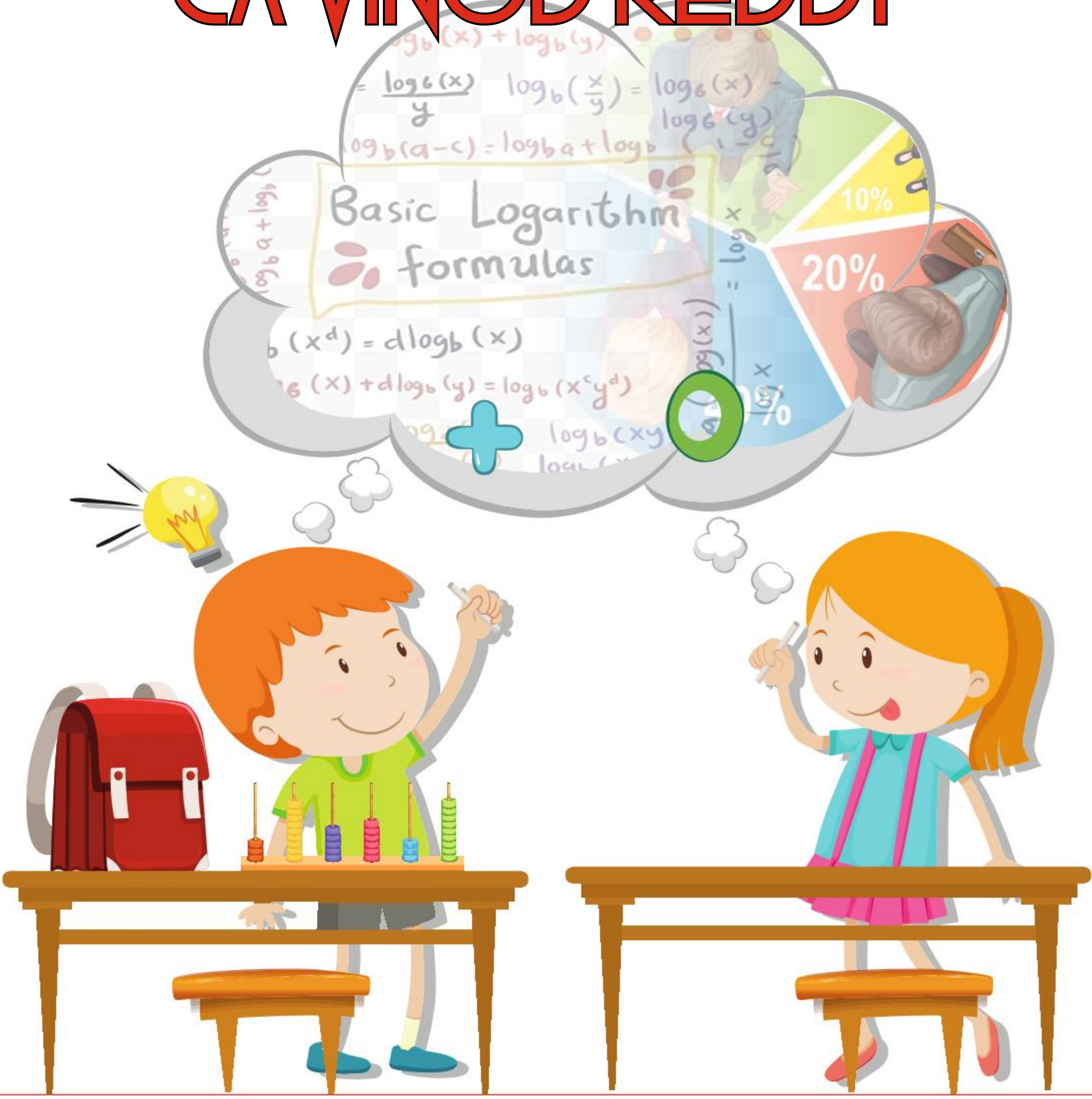

1 What is Ratio?

- Ratio is a fraction used for comparison of 2 or more quantities which are :
  - → of same type
  - → expressed in same unit of measurement
- Ratio can be expressed without unit of measurement

2 Find simplest form of 3.50 : 8.75.

⟹ • Generally ratio is written in its simplest form

- $\frac{3.50}{8.75} = \frac{350}{875} = \frac{14}{35} = \frac{2}{5} = 2 : 5$

∴ simplest form is 2 : 5

3 5:7 can also be written as :

$5k : 7k = 50 : 70 = \frac{5}{k} : \frac{7}{k}$ where $k \neq 0$

All the terms of the ratio can be multiplied or divided by same non-zero number.

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Ratio	It's	Answer
5:7	Duplicate Ratio	25 : 49
8:3	Triplicate Ratio	$8^3 : 3^3$ = 512 : 27
11: 19	Inverse Ratio	19 : 11
64:625	Sub-Duplicate Ratio	$\sqrt{64} : \sqrt{625}$ = 8 : 25
125:27	Sub-Triplicate Ratio	$\sqrt[3]{125} : \sqrt[3]{27}$ = 5 : 3

5 Find compounded ratio of 5:7, a:b, x:y, 9:8

Compounded ratio = (Product of antecedents : Product of consequents)

Answer : $\boxed{45ax : 56by}$

6 3 : 8 : 9 : 11 is a continued ratio.

Ratio of 3 or more terms is known as continued ratio.

My Notes

- In a : b , a = First term = Antecedent
  b = second term = consequent
- simplest form of 81 : 36 is $\frac{81}{36} = \frac{9}{4}$
  = 9 : 4
- a : b can also be written as $am : bm = \frac{a}{m} : \frac{b}{m}$ where $m \neq 0$

7 Ratio of 3 or more terms is known as continued Ratio.

8 : 7 : 11 : 19 is a continued ratio.

8 Ratio is unit free. Ratio can be expressed without unit of measurement

9 First term of the ratio = Antecedent

Second term of the ratio = consequent

10 Find the ratio of 3kg : 35,000 grams

= 3000 gms : 35,000 gms = 3 : 35 = $\left(\frac{3}{35}\right)$

11 a:b can also be written as (ak : bk) or $\left(\frac{a}{k} : \frac{b}{k}\right)$ provided $k \neq 0$

5 : 8 can also written as 10 : 16 = 15 : 24 = 50 : 80 = 2.50 : 4 $= \left(\frac{5}{9} : \frac{8}{9}\right) = \left(\frac{5}{13} : \frac{8}{13}\right)$

12 The order of the terms in a ratio is important.

13 Find simplest form of $2\frac{1}{3} : 3\frac{2}{3}$

$$= \frac{2\frac{1}{3}}{3\frac{2}{3}} = \frac{\frac{7}{3}}{\frac{11}{3}} = \frac{7}{11} = 7 : 11$$

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If the Ratio a:b If	then a:b is called as
a > b	Ratio of Greater inequality
a < b	Ratio of Lesser inequality
a = b	Ratio of Equality

15 Ratio exists only when 2 or more quantities are of same kind/type expressed in same unit of measurement.

16 Find simplest form of $\frac{1}{3} : \frac{1}{8} : \frac{1}{10}$

$\Longrightarrow \frac{1}{3} : \frac{1}{8} : \frac{1}{10} = 80 : 30 : 24 = 40 : 15 : 12$

17 Find simplest form of $\frac{3}{5} : \frac{2}{3} : \frac{8}{5}$

$\Longrightarrow \frac{3}{5} : \frac{2}{3} : \frac{8}{5}$

$= 9 : 10 : 24$

My Notes

Find simplest form of $8\frac{1}{7} : 9\frac{3}{11}$

$$\Longrightarrow \frac{8\frac{1}{7}}{9\frac{3}{11}} = \frac{\frac{57}{7}}{\frac{102}{11}} = \frac{57}{7} \times \frac{11}{102} = \frac{627}{714} = \frac{209}{238}$$

= 209 : 238

18. Ratios are unit - free

Ratio can be expressed without any unit of measurement. example : Ratio of Incomes of A, B, C is 2 : 3 : 8

19.

If a:b = 2:3
b:c = 4:7
c:d = 8:1
Find a:b:c:d, a:d, b:d

a : b = 2 : 3 = 8 : 12
b : c = 4 : 7 = 12 : 21
∴ a : b : c = 8 : 12 : 21
c : d = 8 : 1
a : b : c = 64 : 96 : 168
c : d = 168 : 21
∴ a : b : c : d = 64 : 96 : 168 : 21
∴ a : d = 64 : 21
b : d = 96 : 21 = 32 : 7

20. If Quantity increase or decreases in the ratio a:b then new quantity = b of original quantity = a

∴ New quantity = (original quantity x mutiplying ratio)

where multiplying ratio = (Reciprocal of given ratio)

original quantity = (new quantity x Given ratio)

A's Income ₹ 7 lakhs.
his income changes in the ratio of 21 : 29.
Find his new income ?

⟹ New Income = ₹ old income x multiply ratio
$= ₹ 7 \times \frac{29}{21} = ₹ 9.666666$ lakhs

21. Population of a city is x then it changes in the ratio of p:q then find new population

⟹ New population = Old population x Multiplying Ratio
$= x \times \frac{q}{p} = \left(\frac{qx}{p}\right)$

22.

Inverse ratio of Inverse ratio of a:b is = a : b
Duplicate ratio of sub duplicate ratio of p:q is = p : q
Triplicate ratio of sub triplicate ratio of m:n is = m : n
Sub triplicate ratio Triplicate ratio of x:y is = x : y
Sub duplicate ratio of duplicate ratio of u:v = u : v

23. Find Duplicate ratio of Inverse ratio of 5:7

⟹ Duplicate ratio of 7 : 5 is
= 49 : 25

24. Find Triplicate ratio of sub duplicate ratio of 25:49

⟹ Triplicate ratio of 5 : 7
= 125 : 343

My Notes :

25. **Find compounded ratio of Duplicate ratio of 2:3, Triplicate ratio of 9:4, Sub duplicate ratio of 81:64, sub duplicate ratio of 512:27**

$\Rightarrow \quad 4 : 9 \; , \; 729 : 64 \; , \; 9 : 8 \; , \; \sqrt{512} : \sqrt{27}$

$$\text{compounded Ratio} = \frac{4 \times 729 \times 9 \times \sqrt{512}}{9 \times 64 \times 8 \times \sqrt{27}} = \frac{81 \times 9 \times \sqrt{256 \times 2}}{2 \times \sqrt{9 \times 3} \times 64}$$

$$= \frac{81 \times 9 \times 16 \times \sqrt{2}}{2 \times 3\sqrt{3} \times 64} = \frac{81\sqrt{6}}{8}$$

My Notes :

$$\text{compounded Ratio} = \left(\text{Product of antecedents} : \text{product of consequents} \right)$$

26 **When 4 quantities a,b,c,d are said to be in proportion?**

$\Rightarrow$ • when $a:b = c:d$ then a,b,c,d are said to be in proportion.

when $a:b = c:d$ $\therefore \frac{a}{b} = \frac{c}{d}$ i.e. $ad = bc$

(product of extremes = product of means)

• If $ps = qr$ then p,q,r,s are said to be in proportion.

27 **When 4 quantities a,b,c,d are said to be in continued proportion?**

$\Rightarrow$ when $a:b = b:c = c:d$ then a,b,c,d are said to be continued proportion.

• If $p:q = q:r = r:s$ then p,q,r,s are said to be in continued proportion.

• 2,6,10,30 are in proportion but not in continued proportion.

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4 Quantities	Whether 4 Quantities are in	
	Continued Proportion?	Proportion?
2,6,18,54	Yes	yes
3,8,12,32	NO	Yes
8,24,96,288	NO	yes
8,5,80,45	NO	NO
4,6,9,13.50	Yes	yes

29 **When 3 quantities a,b,c are said to be in proportion?**

• If $a:b = b:c$ then a,b,c are said to be in proportion as well as in continued proportion.

• 10,30,90 are in proportion as well as continued proportion.

30 **If a,b,c,d are in proportion i.e. $\frac{a}{b} = \frac{c}{d}$ then**

Invertendo : $\left(\frac{b}{a} = \frac{d}{c}\right)$

Alternendo : $\left(\frac{a}{c} = \frac{b}{d}\right)$

Componendo : $\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$

Addendo : $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

Dividendo : $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$

Subtrahendo : $\frac{a}{b} = \frac{c}{d} = \left(\frac{a-c}{b-d}\right) = \left(\frac{c-a}{d-b}\right)$

Componendo and Dividendo : $\left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$

If $\frac{50}{60} = \frac{5}{6}$ then

$\frac{50}{60} = \frac{5}{6} = \left(\frac{50+5}{60+6}\right) = \left(\frac{50-5}{60-6}\right)$

31 If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=\frac{i}{j}$ = k, then

As per addendo k = $\left(\frac{a+c+e+g+i}{b+d+f+h+j}\right) = \frac{a}{b} = \left(\frac{a+3c+5e+8g+i}{b+3d+5f+8h+j}\right)$

As per subtrahendo k = $\left(\frac{a+c-e-g+i}{b+d-f-h+j}\right) = \left(\frac{a+100c-10e-25g+100i}{b+100d-10f-25h+100j}\right)$

32 If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$ then, Find value of $\left(\frac{4a+2b-3c}{5b}\right)$

$\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=k$

$a=3k,\ b=4k,\ c=7k$

$\left(\frac{4a+2b-3c}{5b}\right) = \frac{12k+8k-21k}{20k} = \frac{-k}{20k} = -\frac{1}{20}$

$\left(\frac{4a}{12}=\frac{2b}{8}=\frac{3c}{21}\right) = \frac{4a+2b-3c}{12+8-21}$

$\left(\frac{5b}{20}\right) = \frac{4a+2b-3c}{-1}$

$-\frac{1}{20} = \left(\frac{4a+2b-3c}{5b}\right)$

33 Find Fourth Proportional to 8, 12, 20

$\Longrightarrow$ 8, 12, 20, M are in proportion

$8M = 12 \times 20$

$M = 30$ ∴ 4th proportional to 8, 12, 20 is 30.

34 Find mean proportional to 9, 25

$\Longrightarrow$ 9, k, 25 are in proportion ∴ $\frac{9}{k}=\frac{k}{25}$

∴ $k^2 = 9 \times 25$ ∴ $k = 15$

35

4 Quantities in Proportion	Value of k = ?
8, 9, k, 63	$504 = 9k$ ∴ $k = 56$
58, -3k, 28, 85	$4930 = -84k$ ∴ $k = -58.6904$
36, 60, 2k, 98	$3528 = 120k$ ∴ $k = 29.40$
-3k, 86, 25, 63	$-189k = 2150$ ∴ $k = -11.37566$

36 Rules of Indices

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^{-m} = \left(\frac{1}{a^m}\right)$
5. $(a.b)^m = a^m \times b^m$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} = (a^m \times b^{-m})$
7. $a^{1/m} = \sqrt[m]{a}$
8. $[(a^m)^n]^p = a^{mnp}$
9. $(a^{m/n}) = (a^m)^{1/n} = \sqrt[n]{a^m}$
10. If $a^x = a^y$; then $x = y$
11. If $a^m = b^m$; then $a = b$

37 $2x^{1/2} \times 3x^{-1} = ?$ If $x = 4$

$= 2 \times (4)^{1/2} \times 3 \times (4)^{-1}$

$= 2 \times 2 \times 3 \times \frac{1}{4} = 3$

38 $\frac{6ab^2c^3}{2a^2bc^8} =$

$\frac{3 \cdot 6 \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c}{2 \cdot a \cdot a \cdot b \cdot c \cdot c \cdot c \cdot c^5} = \left(\frac{3b}{a \cdot c^5}\right)$

$= 3a^{-1} \cdot b \cdot c^{-5}$

39 $\frac{64 \times \sqrt[3]{128}}{\sqrt[5]{512}} =$

$\frac{2^6 \times (2^7)^{1/3}}{(2^9)^{1/5}} = \frac{2^6 \times 2^{7/3}}{2^{9/5}} = (2)^{6 + \frac{7}{3} - \frac{9}{5}}$

$= (2)^{\frac{90+35-27}{15}} = (2)^{98/15} = \sqrt[15]{2^{98}}$

40 $\frac{4x^{-1}}{x^{-1/3}} =$

$\frac{4 \cdot x^{-1}}{x^{-1/3}} = 4 \cdot (x)^{-1+\frac{1}{3}} = 4 \cdot x^{-2/3}$

$= \left(\frac{4}{x^{2/3}}\right) = \frac{4}{\sqrt[3]{x^2}}$

41 $\frac{2a^{1/2} \times a^{2/3} \times a^{-7/3}}{9a^{-5/3} \times a^{3/2}} = ?$ If $a = 4$

$= \frac{2(4)^{1/2} \times (4)^{2/3} \times (4)^{-7/3}}{9 \times (4)^{-5/3} \times (4)^{3/2}}$

$= \frac{2}{9}(4)^{\frac{1}{2} + \frac{2}{3} - \frac{7}{3} + \frac{5}{3} - \frac{3}{2}}$

$= \frac{2}{9}(4)^{\frac{1}{2} - \frac{3}{2}}$

$= \frac{2}{9}(4)^{-1}$

$= \frac{2}{9} \times \frac{1}{4} = \frac{2}{36} = \frac{1}{18}$

42

① $\frac{(a^m \times a^n \times a^p)}{a^x} = (a)^{(m+n+p-x)}$

② $\frac{64^{1/3}}{1024^{-8/3}} = \frac{(2^6)^{1/3}}{(2^{10})^{-8/3}} = \frac{2^{6/3}}{2^{-80/3}} = (2)^{\frac{6}{3} + \frac{80}{3}} = 2^{86/3}$

$= \sqrt[3]{2^{86}}$

43 $\sqrt[6]{a^{4b} \cdot x^{6} \; (a^{2/3} \cdot x^{-1})^{-b}} = ?$

$= \left[a^{4b} \cdot x^{6} \cdot a^{-2b/3} \cdot x^{b}\right]^{1/6}$

$= \left(a^{4b-\frac{2b}{3}} \cdot x^{6+b}\right)^{1/6} = \left(a^{\frac{10b}{3}} \cdot x^{6+b}\right)^{1/6} = a^{\frac{5b}{9}} (x)^{1+\frac{b}{6}}$

44 $(\sqrt{9})^{7} \times (\sqrt{3})^{-5} = 3^{k}$ then k = ?

$3^{7} \times (3^{1/2})^{-5} = 3^{k}$

$3^{7} \times 3^{-5/2} = 3^{k}$

$(3)^{7-\frac{5}{2}} = (3)^{k}$

$(3)^{9/2} = (3)^{k}$

$\therefore k = 9/2$

$k = 4.50$

45 $\frac{2^5}{2^5} = (2)^{5-5} = 2^{0} = 1 \quad \therefore (\text{Any number})^{0} = 1$

46 $\left(\frac{81x^4}{y^{-8}}\right)^{1/4} = \frac{(81x^4)^{1/4}}{(y^{-8})^{1/4}} = \frac{(3x)^{4\times\frac{1}{4}}}{y^{-8\times\frac{1}{4}}} = \frac{(3x)^{1}}{(y^{-2})} = 3xy^{2}$

47 $\left\{\frac{(3^3)^2 \times (4^2)^3 \times (5^3)^2}{(3^2)^3 \times (4^3)^2 \times (5^2)^3}\right\} = \frac{3^6 \times 4^6 \times 5^6}{3^6 \times 4^6 \times 5^6} = 1$

48 $y^{a-b} \cdot y^{b-c} \cdot y^{c-a} = ?$

$= (y)^{a-b+b-c+c-a} = (y)^{0} = 1$

49 $\left[1-\left\{1-(1-x^2)^{-1}\right\}^{-1}\right]^{-1/2} =$

$= \left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1+\frac{1-x^2}{x^2}\right]^{-1/2}$

$= \left[\frac{1}{x^2}\right]^{-1/2}$

$= (x^{-2})^{-1/2}$

$= x^{1} = x$

50 $\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$

$= \left[(x^n)^{\frac{n^2-1}{n}}\right]^{\frac{1}{(n+1)}}$

$= \left[x^{n\times\frac{(n-1)(n+1)}{n}}\right]^{\frac{1}{n+1}}$

$= x^{(n-1)(n+1)\times\frac{1}{(n+1)}}$

$= (x)^{n-1}$

$= \left(\frac{x^n}{x}\right)$

51 If $a^x = b$, $b^y = c$, $c^z = a$ then $xyz = ?$

$a^x = b$

$Log\, a^x = Log\, b$

$x \cdot Log\, a = Log\, b$

$x = \frac{Log\, b}{Log\, a}$ $\quad y = \frac{Log\, c}{Log\, b}$ $\quad z = \frac{Log\, a}{Log\, c}$

$xyz = \frac{Log\, b}{Log\, a} \times \frac{Log\, c}{Log\, b} \times \frac{Log\, a}{Log\, c} = 1$

$a^x = b$, $b^y = c$, $c^z = a$

∴ $a = b^{1/x}$

∴ $b = c^{1/y}$

$c = a^{1/z}$

$c = (b^{1/x})^{1/z}$

$c = (b)^{1/xz}$

$c = (c^{1/y})^{1/xz}$

$c^1 = (c)^{1/xyz}$

∴ $1 = \frac{1}{xyz}$

∴ $xyz = 1$

52 $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \cdot \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \cdot \left(\frac{x^c}{x^a}\right)^{(c^2+ac+a^2)} = ?$

Pls. Remember: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= (x^{a-b})^{a^2+ab+b^2} \cdot (x^{b-c})^{b^2+bc+c^2} \cdot (x^{c-a})^{c^2+ac+a^2}$

$= x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3} = (x)^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = 1$

53 Log of number consist of 2 parts

Integer Part = characteristic

Fractional Part = Mantissa

54 Log x = characteristic of x + Mantissa of x

$Log\,(ab) = Log\, a + Log\, b$

$Log\, b^a = a \cdot Log\, b$

$Log\, m^{(ab)} = ab\,(Log\, m)$

$Log_m (a/b) = Log_m a - Log_m b$

If $log_b a = k$; then $b^k = a$

If $x^y = z$; then $Log_x z = y$

$Log\,(a)^{-b} = -b \cdot Log\, a$

$Log\,(ab/c) = Log\, a + Log\, b - Log\, c$

$A.log\,(log\, x) = x$

$Log\,(A.log\, x) = x$

$Log_a a = 1$ provided $a \neq 1$

$Log_b a \times Log_c b = Log_c a$

$Log_b a = \left(\frac{Log\, a}{Log\, b}\right)$

$Log_{10} 10 = 1$

$Log_{10} 1 = 0$

$Log_{10} 100 = 2$

$Log_{10} 10{,}000 = 4$

$Log_{10} 1000 = 3$

common base of Logs : 10

Natural base of Logs : e

$Log_m abc = Log_m a + Log_m b + Log_m c$

$\boxed{\text{If } Log_a m = j \text{ then } a^j = m}$

$\boxed{Log\, a^z = z \cdot Log\, a}$

55

$$\frac{\text{Log}_3 8}{\text{Log}_9 16 \times \text{Log}_4 10} = \left[\frac{\frac{\text{Log}8}{\text{Log}3}}{\frac{\text{Log}16}{\text{Log}9} \times \frac{\text{Log}10}{\text{Log}4}}\right] = \frac{\text{Log}2^3}{\text{Log}3} \times \frac{\text{Log}3^2}{\text{Log}2^4} \times \frac{\text{Log}2^2}{\text{Log}10}$$

$$= \frac{3.\text{Log}2}{\text{Log}3} \times \frac{2.\text{Log}3}{4.\text{Log}2} \times 2.\text{Log}2$$

$$= 3.\text{Log}2 = \text{Log}2^3 = \text{Log}8$$

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Log x (where x > 0)

Characteristic of x (Integer part) + **Mantissa of x** (fractional part)

- **If x ≥ 1**: (No. of digits before decimal point - 1)
- **1 > x > 0**: (No. of zeros immediately after decimal point + 1) BAR

57

x	Characteristic of x	x	characteristic of x
56.81	2 − 1 = 1	0.0081	$\bar{3}$
583.2	3 − 1 = 2	3.639	0
81.93	2 − 1 = 1	0.01181	$\bar{2}$
5.81	1 − 1 = 0	13.21	1
13.00	2 − 1 = 1	88888	4
0.008126	$\bar{3}$	33.63	1
0.5826	$\bar{1}$	7.819	0
8.5926	1 − 1 = 0	0.0021	$\bar{3}$

58 **How to find Log x on calculator?**

⟹ Enter x
then press √ 15 times
then deduct 1
then Multiply by 14230.9635
You will get Answer of Log x on calculator

Log 23.81
= 1.376826079889

59 **How to find A.log y on calculator?**

⟹ Enter 'y'
then divide by 14230.9635
then Add 1
then (x =) 15 times

A.Log 1.376826079889
= 23.81

60 How to find a^b on calculator? (Particularly when b is in fractions)

⟹ How to find a^b

1) Enter 'a'
2) $\sqrt{\ }$ 12 times
3) Deduct 1
4) Multiply by 'b'
5) Add 1
6) $\times =$ 12 times

$\sqrt[5]{600} = 600^{1/5} = 600^{0.20}$

$= 3.59731$

61 Common base of Logs is : 10

Natural base of Logs is : e where e = exponential factor

$= 2.7183$ (approx)

62 $\log_{\sqrt{2}} 64 = \frac{\log 64}{\log \sqrt{2}} = \frac{\log 2^6}{\log 2^{1/2}} = \frac{6 \cdot \log 2}{\frac{1}{2} \log 2} = 6 \times 2 = 12$

$\log_{\sqrt{2}} 64 = m \quad \therefore (\sqrt{2})^m = 64 \quad (2^{1/2})^m = 2^6 \quad \therefore 2^{m/2} = 2^6 \quad \therefore \frac{m}{2} = 6$

$\therefore m = 12$

63 $\log_2 \log_2 \log_2 16 = \log_2 \log_2 \left(\frac{\log 16}{\log 2}\right) = \log_2 \log_2 (4)$

$= \log_2 \left[\frac{2 \log 2}{\log 2}\right] = \log_2 2 = 1$

64 $\log_9 (1/3) = \frac{\log (1/3)}{\log 9} = \frac{\log 3^{-1}}{\log 3^2} = \frac{-1 \cdot \log 3}{2 \cdot \log 3} = -\frac{1}{2} = -0.50$

65 $\log_{16} 32^{-8} = \frac{\log (32)^{-8}}{\log (16)} = \frac{\log (2^5)^{-8}}{\log (2)^4} = \frac{\log (2)^{-40}}{\log (2)^4}$

$= \frac{-40 \log 2}{4 \log 2} = -10$

66 Log x = (m + n) ; Log y = (m - n); then

$\log \left(\frac{10x}{y^2}\right) = \log 10x - \log y^2$

$= \log 10 + \log x - 2 \log y$

$= 1 + m + n - 2(m - n)$

$= 1 + m + n - 2m + 2n$

$= (-m + 3n + 1) = (3n - m + 1)$

67 $2 \log 5 + \log 8 - (1/2) \log 4 =$

$= \log 5^2 + \log 8 - \log (4)^{1/2}$

$= \log 25 + \log 8 - \log 2$

$= \log \left(\frac{25 \times 8}{2}\right) = \log 100 = 2.00$

68 $\sqrt[4]{729 \times \sqrt[3]{9^{-1} \times 27^{-4/3}}} = ?$

$= \left[3^6 \times \left(\frac{1}{3^2} \times \frac{1}{3^4}\right)^{1/3}\right]^{1/4} = \left[3^6 \times \left(\frac{1}{3^6}\right)^{1/3}\right]^{1/4} = \left[3^6 \times (3^{-6})^{1/3}\right]^{1/4}$

$= \left[3^6 \times 3^{-2}\right]^{1/4} = \left[3^4\right]^{1/4} = 3^1 = 3.00$

69 $\log_{2\sqrt{2}} 64 = ?$

$= \frac{\log 64}{\log 2\sqrt{2}} = \frac{\log 2^6}{\log (2 \times 2^{1/2})} = \frac{\log 2^6}{\log 2^{3/2}} = \frac{6 \cdot \log 2}{\frac{3}{2} \log 2} = 6 \times \frac{2}{3} = 4$

70 Find 4^{th} proportional to 2/3, 3/7, 4.

$\Longrightarrow \frac{2}{3}, \frac{3}{7}, 4, m$

$\therefore \frac{2m}{3} = \frac{3}{7} \times 4$

$\therefore m = \frac{3}{2} \times 4 \times \frac{3}{7}$

$m = \frac{36}{14} = \frac{18}{7}$

$\therefore$ 4th prop. to $\frac{2}{3}, \frac{3}{7}, 4$ is $\frac{18}{7}$

71 If $2^x = 3^y = 6^{-z}$; then $(1/x) + (1/y) + (1/z) = ?$

$2^x = 3^y = 6^{-z} = k$

$x \log 2 = y \log 3 = -z \log 6 = \log k$

$x = \frac{\log k}{\log 2}, \quad y = \frac{\log k}{\log 3}, \quad z = \frac{-\log k}{\log 6}$

$= \frac{\log 2}{\log k} + \frac{\log 3}{\log k} - \frac{\log 6}{\log k}$

$= \frac{\log 2 + \log 3 - \log 6}{\log k} = \frac{\log (2 \times 3) - \log 6}{\log k}$

$= (\log 6 - \log 6)/\log k = \text{zero}.$

72 Find in what ratio will the total wages of the workers of a factory be increased or decreased if there is reduction in no. of workers in the ratio of 17:12 and increment in wage rate per worker in the ratio of 24:29

$\Longrightarrow$

	old	New
No of workers	x	$x \times \frac{12}{17} = \frac{12x}{17}$
wage rate per worker	y	$y \times \frac{29}{24} = \frac{29y}{24}$
Total wages	xy	$\frac{12x}{17} \times \frac{29y}{24} = \frac{29xy}{34}$

$\left(\frac{\text{old Total wages}}{\text{New Total wages}}\right) = \frac{xy}{\frac{29xy}{34}}$

$= 34 : 29$

73 What least number must be added to each one of 6, 14, 18, 38 to make them in proportion

a. 5   b. 3   ~~c. 2~~   d. 4

8, 16, 20, 40 are in proportion

74 The incomes of X and Y are in the ratio of 3:2 and their expenditures are in the ratio of 5:3. If each saves ₹ 1500 then incomes of X and Y resp. are :

⇒ Incomes of X, Y are $3x, 2x$

$$\frac{3x-1500}{2x-1500} = \frac{5}{3}$$

$9x - 4500 = 10x - 7500$

$\boxed{3000 = x}$

Incomes of X, Y are 9000, 6000

75 In a sugar solution of 300 gms, the proportion of sugar is 40%. How much sugar should be added to make it 50%

⇒ Sugar = 300 × 40% = 120 gms

Let x gms of sugar be added

$50\%(300 + x) = (120 + x)$

$150 + 0.50x = 120 + x$

$30 = 0.50x$

$\boxed{x = 60 \text{ gms}}$

76 A mixture contains milk and water in the ratio of 5:1. On adding 5 litres of water, the ratio of milk to water becomes 5:2. The quantity of milk in the original mixture is :

⇒ original mixture ⇒ milk : water = $5x : x$

$$\left(\frac{5x}{x+5}\right) = \frac{5}{2}$$

∴ $10x = 5x + 25$

$5x = 25$

$x = 5$ litres.

Qty of milk in original mixture 25 litres

77 If the denominator of a fraction exceed the numerator by 8. If numerator and denominator are both increased by 5, then fraction becomes 3/5. Find the original fraction.

⇒ $\left(\frac{x}{x+8}\right)$ be original fraction.

$$\left(\frac{x+5}{x+13}\right) = \frac{3}{5}$$

∴ $5x + 25 = 3x + 39$

$2x = 14$

$x = 7$

∴ original Fraction is $7/15$

78 If $\text{Log}_{3/2} x = 3$, Find value of x

∴ $\left(\frac{3}{2}\right)^3 = x$

∴ $x = \left(\frac{3^3}{2^3}\right) = \frac{27}{8}$

If $\text{Log}_b a = k$ then $b^k = a$

79 $\text{Log}_{(1/9)} 243 = x$. Find x

⇒ $\left(\frac{1}{9}\right)^x = 243$ ∴ $(3)^{-2x} = (3)^5$

$\left(\frac{1}{3^2}\right)^x = 3^5$ ∴ $-2x = 5$

$\left(\frac{1}{3}\right)^{2x} = (3)^5$ ∴ $x = -5/2$

OR

$$x = \frac{\log 243}{\log(1/9)}$$

$$x = \frac{\log 3^5}{\log 3^{-2}} = \frac{5 \cdot \log 3}{-2 \cdot \log 3}$$

$x = (-5/2)$

80 Log x^3 - 2 Log x - 2 = 0. Find x

$\Longrightarrow$ $3 \cdot \text{Log}\, x - 2 \,\text{Log}\, x = 2$

$\text{Log}_{10} x = 2$

$\therefore 10^2 = x$

$\therefore x = 100$

81 $\text{Log}_a 3 = 2$, $\text{Log}_b 8 = 3$ then $\text{Log}_b a = ?$

$\Longrightarrow$ $\text{Log}_a 3 = 2$ $\therefore a^2 = 3$, $a = 3^{1/2}$

$\text{Log}_b 8 = 3$ $\therefore b^3 = 8$, $b = 2$

$\text{Log}_b a = \frac{\text{Log}\, a}{\text{Log}\, b} = \frac{\text{Log}\, 3^{1/2}}{\text{Log}\, 2} = \frac{\frac{1}{2}\text{Log}\, 3}{\text{Log}\, 2} = \frac{\text{Log}\, 3}{\text{Log}\, 2^2} = \frac{\text{Log}\, 3}{\text{Log}\, 4} = \text{Log}_4 3$

82 If 2 Log a + 3 Log b - 2 = 0 then a^2b^3 = ?

$\Longrightarrow$ $2\,\text{Log}\, a + 3\,\text{Log}\, b = 2$

$\text{Log}\, a^2 + \text{Log}\, b^3 = 2$

$\text{Log}_{10}(a^2 b^3) = 2$

$\therefore 10^2 = a^2 b^3$

$\therefore (a^2 b^3) = 100$

83 $\text{Log}_2 [\text{Log}_2 \{\text{Log}_3 (\text{Log}_3 27^3)\}]$

$= \text{Log}_2[\text{Log}_2\{\text{Log}_3(9)\}] = \log_2[\text{Log}_2 2] = \log_2[1] = 0$

84 2 numbers are in the ratio of 3:4. If 6 is added to each term then the new ratio will be 4:5 then the numbers are

$\Longrightarrow$ Let 2 Numbers be $3x$, $4x$

$\frac{3x+6}{4x+6} = \frac{4}{5}$ $\therefore 15x + 30 = 16x + 24$

$\therefore 6 = x$

$\therefore x = 6$

$\therefore$ Numbers are $= 18, 24$

85 The sub-duplicate ratio of 1250:50 is :

simplest form of 1250 : 50 = 25 : 1 Answer : (5 : 1)

86 Dhrish earns ₹ 2,780 in 7 hrs and Vinod earns ₹ 990 in 12 hrs. Ratio of their earning per hour is :

$= \left(\frac{2780/7}{990/12}\right) = \frac{2780}{7} \times \frac{12}{990} = \frac{1112}{231} = (1112 : 231)$

87 P, Q, R are 3 cities. The ratio of avg. temp. of P, Q is 11:12 and that of P, R is 9:8. Find the ratio of avg temp. of Q:R.

$\Longrightarrow$ P : Q = 11 : 12 = 99 : 108

P : R = 9 : 8 = 99 : 88

$\therefore$ P : Q : R = 99 : 108 : 88

Q : R = 108 : 88 = 27 : 22

88 If 2s : 3t is the duplicate ratio of (2s-p) : (3t-p) then

a. $p^2 = 6st$ b. p = 6st c. 2p = 3st d. None of these

$\Longrightarrow$ $\frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$

$\frac{2s}{3t} = \frac{4s^2 - 4ps + p^2}{9t^2 - 6pt + p^2}$

$18t^2 s - \cancel{12pts} + 2sp^2 = 12s^2 t - \cancel{12pst} + 3p^2 t$

$18t^2 s - 12s^2 t = 3p^2 t - 2sp^2$

$6st(3t - 2s) = p^2(3t - 2s)$

$\therefore p^2 = 6st$

89 **If A = B/2 = C/5; then A:B:C is :**

$$\Rightarrow \left(\frac{A}{1} = \frac{B}{2}\right) = \frac{C}{5} \qquad \therefore A:B:C = 1:2:5$$

$$A:B = 1:2,\quad B:C = 2:5$$

90 **Log5 = 0.6990, Log 3 = 0.4771 then Log (50/300) = ?**

$$\Rightarrow \text{Log}\left(\frac{50}{300}\right) = \text{Log}\,50 - \text{Log}\,300$$
$$= 1.6990 - 2.4771$$
$$= -0.7781$$

$$\text{Log}\,(50/300)$$
$$= \text{Log}\,(0.1666666)$$
$$= -0.7781$$

91 **Log 2 = x; Log 3 = y; then Log 60 = ?**

$$\Rightarrow \text{Log}\,(60) = \text{Log}\,(3 \times 2 \times 10)$$
$$= \text{Log}\,3 + \text{Log}\,2 + \text{Log}\,10$$
$$= (x + y + 1)$$

92 **Log (1/81) to the base 9 is equal to :**

$$\Rightarrow = \frac{\text{Log}\,(1/81)}{\text{Log}\,9} = \frac{\text{Log}\,(9^{-2})}{\text{Log}\,9} = \frac{-2\,\text{Log}\,9}{\text{Log}\,9} = -2$$

93 **$\bar{4}.5671 + 7.8253 = ?$**

$$= (-4 + 0.5671) + 7.8253$$
$$= -3.4329 + 7.8253 = 4.3924$$

$$\bar{8}.2386 + \bar{3}.2629$$
$$= -8 + 0.2386 - 3 + 0.2629$$
$$= -10.4985$$

94 $\sqrt[(a+b)]{\dfrac{x^{a^2}}{x^{b^2}}} \cdot \sqrt[(b+c)]{\dfrac{x^{b^2}}{x^{c^2}}} \cdot \sqrt[(c+a)]{\dfrac{x^{c^2}}{x^{a^2}}}$

$$= \left(x^{a^2-b^2}\right)^{\frac{1}{a+b}} \cdot \left(x^{b^2-c^2}\right)^{\frac{1}{b+c}} \cdot \left(x^{c^2-a^2}\right)^{\frac{1}{c+a}}$$

$$= x^{(a+b)(a-b) \times \frac{1}{(a+b)}} \cdot x^{(b-c)(b+c) \times \frac{1}{(b+c)}} \cdot x^{(c-a)(c+a) \times \frac{1}{(c+a)}}$$

$$= x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$$

$$= x^{a-b+b-c+c-a}$$

$$= x^{0} = 1$$